A robust measurement protocol for Bohmian velocities

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A protocol for measuring Bohmian velocities using two position measurements (each one modeled by a Positive Operator Valued Measure) separated by a finite time-interval is presented. The protocol is very accurate and robust so long as the first measurement uncertainty divided by the finite time-interval between measurements is much larger than the Bohmian velocity, and the system evolves under flat potential between measurements. The difference between the Bohmian velocity of the unperturbed state and the measured one is predicted to be much smaller than 1% in a large range of parameters. Counter-intuitively, the measured velocity is that at the final time and not a time-averaged value between measurements.

The velocity of a classical object, requiring two position measurements, is trivially implemented in many apparati which control our daily activity. On the contrary, in the quantum world, the first position measurement implies a perturbation on the quantum system so that the knowledge of the velocity without perturbation is hardly accessible. One can minimize the back-action of the measurement on the system using the pre- and post-selected *weak* measurement, developed by Aharonov, Albert and Vaidman (AAV) [1] more than two decades ago and receiving increasing attention [2–7] nowadays. As a relevant example, the spatial distribution of *velocities* of relativistic photons in a double slit scenario has been measured, and the associated quantum trajectories reconstructed [6]. Recently, Wiseman clarified the physical meaning of such weak measurement of *velocities* in non-relativistic scenarios [3]. Using the weak AAV value [1], he showed that the velocity distribution constructed from an arbitrary pre-selected state and a post-selected position eigenstate with an infinitesimal temporal separation between measurements corresponds exactly to the Bohmian velocity of the unperturbed state [8].

Naively, one could think that such velocities distributions pushed orthodox quantum mechanics towards a contradiction because of the exact measurement of the position and momentum of an unperturbed state. Obviously, such contradiction is not present because the mentioned experimental and theoretical velocity distributions are computed from an *ensemble* of identical systems (not from an individual one). In other words, Wiseman's results [3] can be interpreted either as the orthodox hydrodynamic velocity [9] or as a genuine measurement of the Bohmian velocity [10]. Both velocities are mathematically equivalent. Since these *ensemble* Bohmian velocities are free from fundamental controversies, one can envision their use (instead of the wave function) as a practical interpretative tool for studying general quantum phenomena [11, 12]. However, the practical conditions for measuring Bohmian velocities in a laboratory are different from the idealized scenario studied by Wiseman [3]. First, the *weak* measurement in a laboratory can be outside of the linear-response regime assumed in the AAV development [13]. Second, position measurements have a small but finite uncertainty, meaning that the post-selected state is not an exact position eigenstate. Identically, the time-separation between measurements must be finite. In this letter, using the Positive Operator Valued Measure (POVM) framework [13] to deal with two position measurements separated by a finite time, we extend the original Wiseman's conclusions about the measurable Bohmian velocities into practical laboratory conditions (free from previous idealized assumptions).

The POVM appears as a natural modeling of a measuring process [14] when the laboratory is divided into the quantum system and the rest (including the measuring apparatus). Thus, the perturbation of the state due to the measurement of the first position x_w can be defined through the Gaussian measurement (Krauss) operators:

$$\hat{W}_w = C_w \int dx e^{-\frac{(x_w-x)^2}{2\sigma_w^2}} |x\rangle \langle x| , \tag{1}$$

where σ_w is the experimental uncertainty. The measured position x_w belongs to the set $\mathfrak{M}$ of all possible measurement outputs of the apparatus. For simplicity, we assume $\mathfrak{M} \equiv \mathbb{R}$ in a 1D system, being the extension to the 3D spatial domain trivial. Then, the normalization coefficient $C_w = (\sqrt{\pi}\sigma_w)^{-1/2}$ is fixed by the condition $\int dx_w \hat{W}_w^\dagger \hat{W}_w = I$. Due to the unavoidable uncertainty on any position measurement, we consider an equivalent operator for the second position measurement of x_s:

$$\hat{S}_s = C_s \int dx e^{-\frac{(x_s-x)^2}{2\sigma_s^2}} |x\rangle \langle x| . \tag{2}$$

From a large set of measured positions, x_w at time t_w and x_s at $t_s = t_w + \tau$, we construct the experimental velocity as:

$$v_e(x_s, t_s) = \tau^{-1} E[(x_s - x_w)|x_s], \tag{3}$$

being $E[(x_s - x_w)|x_s]$ the ensemble average of the distance $x_s - x_w$, conditioned to the fact that x_s is effectively measured. Since $E[x_s|x_s] = x_s$, the theoretical computation of the velocity v_e does only requires evaluating $\mathrm{E}[x_w|x_s]$ using standard probability calculus,

$$\mathrm{E}[x_w|x_s] = \frac{\int dx_w x_w P(x_w \cap x_s)}{P(x_s)}, \tag{4}$$

with $P(x_w \cap x_s)$ the joint probability of the sequential measurements of x_w and x_s, and $P(x_s)$ of x_s. Both probabilities can be computed from the Born rule, using (1) and (2), as:

$$P(x_w \cap x_s) = \langle\Psi|\, \hat{W}_w^\dagger U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \hat{W}_w \,|\Psi\rangle \tag{5}$$

$$P(x_s) = \int dx_w P(x_w \cap x_s). \tag{6}$$

being $|\Psi(t_w)\rangle \equiv |\Psi\rangle$ the initial state. Strictly speaking, contrarily to the AAV expression [1], we are using a weak measurement without post-selection. The final state of the system (determined by the time-evolution of the initial state $|\Psi\rangle$ and the measurement processes) has no relevant effect when computing (5) and (6).

Let us now analyze $P(x_s)$ in detail by substituting Eq. (1) and (2) into Eq. (6). Then, we have

$$P(x_s) = C_w^2 \iiint dx_w dx' dx'' e^{-\frac{(x_w-x')^2}{2\sigma_w^2}} e^{-\frac{(x_w-x'')^2}{2\sigma_w^2}} \times \\ \times \langle\Psi|\, x'\rangle \langle x'|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \,|x''\rangle \langle x''|\, \Psi\rangle\,. \tag{7}$$

Integrating over x_w and using Eq. (2), we can rewrite Eq. (7) as:

$$P(x_s) = C_s^2 \iint dx' dx'' \,\langle\Psi|\, x'\rangle\, e^{-\frac{(x'-x'')^2}{4\sigma_w^2}} \,\langle x''|\, \Psi\rangle \times \\ \times \left(\int dx e^{-\frac{(x_s-x)^2}{\sigma_s^2}} \,\langle x'|\, U_\tau^\dagger \,|x\rangle \,\langle x|\, U_\tau \,|x''\rangle \right). \tag{8}$$

For a particle of mass m that evolves under a flat potential during τ, we can evaluate $\langle x|\, U_\tau \,|x'\rangle$ using [15]

$$\langle x|\, U_\tau \,|x'\rangle = (i\pi(2\hbar\tau/m))^{-1/2}\, e^{\frac{i(x-x')^2}{(2\hbar\tau/m)}}. \tag{9}$$

Substituting Eq. (9) into (8) and solving the integral between parenthesis, we have

$$P(x_s) = \iint dx' dx'' e^{-\frac{(x'-x'')^2}{4\sigma_w^2}} e^{-\left(\frac{\sigma_s m}{2\hbar\tau}\right)^2 (x'-x'')^2} \times \\ \langle\Psi|\, x'\rangle \,\langle x'|\, U_\tau^\dagger \,|x_s\rangle \,\langle x_s|\, U_\tau \,|x''\rangle \,\langle x''|\, \Psi\rangle\,. \tag{10}$$

One easily realizes that the probability in (10) can be computed as $P(x_s) = \langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau^\dagger \,|\Psi\rangle$ when the following limit is satisfied,

$$\frac{\sigma_w}{\tau} \gg \frac{\hbar}{m\sigma_s}. \tag{11}$$

Let us emphasize that this condition, includes Wiseman's result [3] as a particular case: $\sigma_w \to \infty$, $\sigma_s \to 0$ and $\tau \to 0$. Our development will justify the effective measurement of the Bohmian velocity (up to a negligible error) for a broad range of σ_w, σ_s and τ.

Identical steps can be done for the evaluation of $\int dx_w x_w P(x_w \cap x_s)$ in Eq. (4). The only difference resides on the integration on x_w, which in this case gives $(x'+x'')/2 \exp[-(x'-x'')^2/4\sigma_w^2]$. Using $\int dx\, x\, |x\rangle \langle x| = \hat{x}$, under the limit (11), we obtain $\int dx_w x_w P(x_w \cap x_s) = \mathrm{Re}(\langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \hat{x} \,|\Psi\rangle)$. Finally, we can rewrite Eq. (4) as:

$$\mathrm{E}[x_w|x_s] = \frac{\mathrm{Re}(\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \hat{x}|\Psi\rangle)}{\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau|\Psi\rangle}. \tag{12}$$

Next, we define the following (averaged) position $\bar{x}_s = \langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s \hat{x} U_\tau \,|\Psi\rangle \,/\, \langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \,|\Psi\rangle$, so that using Eq. (12) and the commutator $[U_\tau, \hat{x}]$, we get:

$$\bar{x}_s - \mathrm{E}[x_w|x_s] = \frac{\mathrm{Re}(\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s [U_\tau,\hat{x}]|\Psi\rangle)}{\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau|\Psi\rangle}, \tag{13}$$

without any reference to $\hat{W}_w$. To further develop Eq. (13), we evaluate the commutator $[U_\tau, \hat{x}]$ using the Maclaurin series for U_τ:

$$[U_\tau, \hat{x}] = \sum_{n=1}^{\infty} \frac{(-i)^n \tau^n}{n! \hbar^n} [\hat{H}^n, \hat{x}], \tag{14}$$

where $\hat{H} = \hat{p}^2/2m + V$ is the system Hamiltonian with V a flat potential at the spatial region where the wave function is different from zero during the time between measurements. No restriction on V for other regions and times. Given two operators $\hat{A}$ and $\hat{B}$, it can be proven that $[\hat{A}^n, \hat{B}] = \sum_{j=1}^{n} \hat{A}^{j-1} [\hat{A}, \hat{B}] \hat{A}^{n-j}$. Then, being $[\hat{H}, \hat{x}] = -i\hbar/m\hat{p}$ and $[\hat{H}, \hat{p}] = 0$, the commutator $[\hat{H}^n, \hat{x}]$ gives:

$$[\hat{H}^n, \hat{x}] = -\frac{i\hbar n}{m} \hat{p} \hat{H}^{n-1}, \tag{15}$$

and substituting Eq. (15) into Eq. (14) we obtain:

$$[U_\tau, \hat{x}] = -\frac{\tau}{m} \hat{p} U_\tau, \tag{16}$$

without considering the limit $\tau \to 0$. Using Eq. (16) and the definition (2), a straightforward calculation for the numerator of Eq. (13) gives:

$$\mathrm{Re}(\langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s [U_\tau, \hat{x}] \,|\Psi\rangle) \equiv \tau \bar{J}(x_s, t_s) = \\ \tau C_s^2 \int dx J(x, t_s) \exp[-(x_s - x)^2/\sigma_s^2], \tag{17}$$

where $J(x, t_s)$ is the standard quantum current probability density [11]. Similarly, we define $\langle\Psi|\, U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \,|\Psi\rangle = C_s^2 \int dx |\Psi(x, t_s)|^2 \exp[-(x_s - x)^2/\sigma_s^2] \equiv |\bar{\Psi}(x_s, t_s)|^2$ for the denominator. Finally, the velocity, defined as Eq. (13) divided by τ, gives:

$$\bar{v}(x_s, t_s) = \frac{\bar{x}_s - E[x_w|x_s]}{\tau} = \frac{\bar{J}(x_s, t_s)}{|\bar{\Psi}(x_s, t_s)|^2}. \tag{18}$$

This expression is just the Gaussian-spatially-averaged current density $\bar{J}(x_s, t_s)$ inside a tube of diameter σ_s divided by the corresponding Gaussian-spatially-averaged probability $|\bar{\Psi}(x_s, t_s)|^2$.

Whether or not the Gaussian-spatially-averaged value (18) is identical to the Bohmian velocity depends on

the measuring apparatus resolution, i.e. σ_s, and the de Broglie wavelength λ associated to $|\Psi\rangle$. Under the limit

$$\sigma_s < \lambda, \tag{19}$$

one can assume $\Psi(x,\tau) \approx \Psi(x_s,t_s)$ for $x \in [x_s-\sigma_s, x_s+\sigma_s]$, so that $\bar{\Psi}(x_s,t_s) \approx \Psi(x_s,t_s)$. Identically, $\bar{J}(x_s,t_s) \approx J(x_s,t_s)$ and $\bar{x}_s = x_s$. Then, Eq. (18) directly recovers the Bohmian velocity $\bar{v}(x_s,t_s) \approx v$ with:

$$v \equiv v(x_s,t_s) = \frac{J(x_s,t_s)}{|\Psi(x_s,t_s)|^2}. \tag{20}$$

Let us mention that the consideration $\sigma_s \approx \lambda$ and the momentum $p = h/\lambda$ implies $\hbar/(m\sigma_s) \approx v$ in the limit (11).

Let us now compute the velocity variance. Since x_s and τ are fixed in Eq. (3), $var(v_e) = var(x_w)/\tau^2$. Thus, $var(x_w) = E[x_w^2|x_s] - (E[x_w|x_s])^2$ where $E[x_w|x_s]$ defined in Eq. (4) is obtained from Eq. (20). The evaluation of $\int dx_w x_w^2 P(x_w \cap x_s)$ follows identical steps as in the computation of $P(x_s)$, where again the only difference resides in the integral in x_w that now gives $(\sigma_w^2/2 + (x'+x'')^2/4)\exp[-(x'-x'')^2/4\sigma_w^2$. Using again $\int dx\, x\,|x\rangle\langle x| = \hat{x}$ and $\int dx\, x^2\,|x\rangle\langle x| = \hat{x}^2$, the final result, under the limit (11), is:

$$\mathrm{E}[x_w^2|x_s] = \tfrac{1}{2}\sigma_w^2 + \tfrac{1}{2}\frac{\mathrm{Re}(\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \hat{x}^2|\Psi\rangle)}{\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau|\Psi\rangle} + \tfrac{1}{2}\frac{\mathrm{Re}(\langle\Psi|\hat{x}U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau \hat{x}|\Psi\rangle)}{\langle\Psi|U_\tau^\dagger \hat{S}_s^\dagger \hat{S}_s U_\tau|\Psi\rangle}, \tag{21}$$

which finally gives [16]

$$var(v) = \frac{\sigma_w^2}{2\tau^2} + \tfrac{2}{m}Q_B(x_s) + O\left(\tfrac{\hbar}{m\tau}\right), \tag{22}$$

where $Q_B(x_s)$ is the (local) Bohmian quantum potential [8, 11]. Under the limits (11) and (19), the term $\sigma_w^2/(2\tau^2)$ in Eq. (22) will be orders of magnitude greater than the other two. For an experimentalist, this means that the presence of the quantum potential on the spatial fluctuations of Eq. (22) will be hardly accessible, and that $var(v)$ provides basically the value σ_w of the apparatus. Such variance can be used to evaluate the number N of measurements needed to obtain (20) with a given error $\varepsilon(N)$, using the well know result from the probability calculus that $\varepsilon(N) = \sqrt{var(v)}/\sqrt{N} \approx \sigma_w/(\tau\sqrt{2N})$.

In order to test how robust (i.e. how independent of σ_w, σ_s and τ) is the possibility of measuring the Bohmian velocity in a laboratory, we compute the (local) error $\varepsilon_w(x_s) \equiv |v_e(x_s) - \bar{v}(x_s)|$. The calculation is simply achieved expanding $\exp[-(x'-x'')^2/4\sigma_w^2]$ in Taylor series by keeping the first three terms and repeating the above calculations for $P(x_s)$ and $P(x_w \cap x_s)$ (notice that the previous calculations are equivalent to an order zero Taylor expansion) obtaining [16]:

$$\varepsilon_w(x_s) = \frac{\tau\hbar^2}{4m^2\sigma_w^2}\left|\frac{2(1-\tau\partial_x v)\partial_x\rho - \tau\rho\partial_x^2 v}{\rho + \frac{\tau^2\hbar^2}{4m^2\sigma_w^2}\partial_x^2\rho}\right|, \tag{23}$$

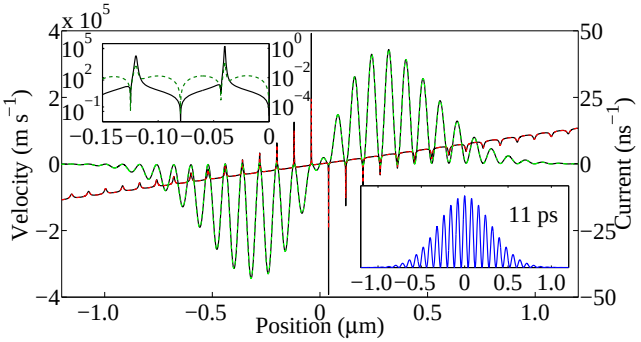

FIG. 1. (Color online) Velocity distribution (v black solid line and v_e red dashed line) and quantum current density (J black solid line and J_e green dashed line) for an electron in a double slit experiment at $t_s = 11$ps, $\sigma_s = 0.2$nm and $\sigma_w = 150$nm. Top inset, total error $\varepsilon_s(x_s) + \varepsilon_w(x_s)$ for the velocity (black solid line) and current (green dashed line). Bottom inset, $|\Psi|^2$ at $t_s = 11$ps.

where $\rho = |\psi(x_s,t_s)|^2$. We further define the measuring apparatus error $\varepsilon_s(x_s) \equiv |v(x_s) - \bar{v}(x_s)|$ deriving from the requirement (19), and a calculation similar to the previous one based on Taylor expansion gives [16]:

$$\varepsilon_s(x_s) = \sigma_s^2\left|\frac{\frac{2}{\tau}\partial_x\rho + (2\partial_x\rho - \rho\partial_x)\partial_x v}{4\rho + \sigma_s^2\partial_x^2\rho}\right|. \tag{24}$$

It is worth noticing that, by construction, the total error $\varepsilon(x_s) \equiv |v(x_s) - v_e(x_s)|$ accomplishes $\varepsilon(x_s) \leq \varepsilon_s(x_s) + \varepsilon_w(x_s)$.

Finally, we observe that the same set of measured values x_w and x_s can be used to define an experimental current density $J_e = \tau^{-1}[P(x_s)x_s - \int dx_w x_w P(x_w \cap x_s)]$. We only have to change how the measured data is analyzed. All the previous calculations can be then repeated for the current in a similar way.

As a numerical test of our prediction, we consider an electron passing through a double slit. For simplicity, the time evolution of two 1D initial Gaussian wave-packets with zero central momenta and central positions separated a distance of 100 nm are explicitly simulated. This corresponds roughly to the evolution of the quantum state after crossing the double-slit at $t = 0$s. From Fig. 1 it is evident that the agreement between the exact Bohmian velocity v in (20) and the v_e [numerically evaluated from (3), (4), (5) and (6) without any limit or approximation] is excellent. In Fig. 2, we plot the normalized value of the error $\varepsilon_w(x_s)$ integrated over x_s as $\varepsilon_w = (\int dx_s \varepsilon_w(x_s)^2 / \int dx_s v(x_s)^2)^{1/2}$. The main conclusion extracted from Fig. 2 is that a large set of parameters (large σ_w/τ values) allows a very accurate measurement of the Bohmian velocity, justifying the robustness of our proposal.

To conclude, we emphasize some relevant issues. First, we have shown theoretically and numerically that the Bohmian velocity of an unperturbed state under general laboratory conditions can be obtained from two POVM

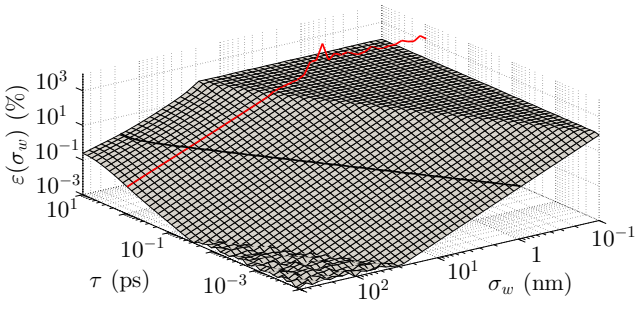

FIG. 2. (Color online) Relative error ε_w integrated over all positions x_s as a function of σ_w and τ for $\sigma_s = 0.2$nm for the numerical test represented in Fig. 1. Black line bounds the region for $\varepsilon(\sigma_w) \leq 1\%$ and red line is the analytical error for the value $\tau = 1$ps.

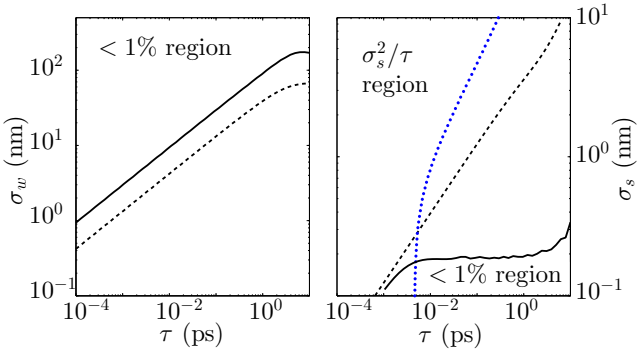

FIG. 3. (Color online) Left inset, region of relative error $\varepsilon_w < 1\%$ and right inset region of relative error $\varepsilon_s < 1\%$. Solid lines are the boundaries for the velocity and dashed line are the boundaries for the quantum current. Dotted line bounds the σ_s^2/τ region.

measurements separated by a finite τ. Unlike the results derived from the AAV formulation [1], the limits (11) and (19) provide a simple quantitative explanation of the experimental conditions for an accurate and robust measurement of the Bohmian velocity.

On the other hand, the error $\varepsilon_s(x_s)$ in (24) has a term that diverges as σ_s^2/τ, meaning that a τ close to zero will produce an inaccurate measurement of the velocity for finite σ_s. This regime is reported in the right inset of Fig. 3. Roughly speaking, for $\tau \to 0$, the wave packet moves a distance $v\tau$. When $v\tau < \sigma_s$ the measured position x_s has no relation to the velocity. We emphasize again that Wiseman's result [3] does not suffer from this inaccuracy because he considers, both, $\sigma_s \to 0$ and $\tau \to 0$.

A closer look at the expressions (23) and (24) shows that the error diverges when ρ has oscillations with minima tending to zero. This can be clearly seen in Fig. 1 where the highest peak of the velocity corresponds to a minimum of ρ very close to zero. This situation is reversed when we evaluate the current J. In fact, in these critical points, $J \to 0$ and even the corresponding errors become very small. In Fig. 3 it is evident the shift of the $< 1\%$ region due to this error reduction.

Perhaps, the most surprising feature of our protocol is that a local (in time and position) Bohmian velocity can be measured with a large temporal separation between measurements, while one would expect a time-averaged value. This is highly counter-intuitive because we are in a scenario where the time-evolving interferences imply large acceleration of the Bohmian particle in order to rapidly avoid the nodes of the wave function.

Finally, another relevant result is that the accuracy of the Bohmian velocity is obtained at the prize of increasing the dispersion on x_w (as seen in Eq. (22) for large σ_w). Therefore, the fact that we can obtain the Bohmian velocity is not because the system remains unperturbed after one position measurement, rather because of the ability of the ensemble average done in the x_w integrals on Eq. (5) and Eq. (6) to compensate for the different perturbations. The fact that a very large perturbation of the state is fully compatible with a negligible error can be easily seen in our numerical data. The *measured* state is roughly equal to the product of the unperturbed wave function (whose support is $L \approx 2000$nm at time $t_w = 11$ps in Fig. 1) by a Gaussian function centered at the measured position with a dispersion equal to σ_w (for example, $\sigma_w \approx 150$nm for $\tau = 1$ps in Fig. 2). Even for $\sigma_w << L$ (i.e. a large perturbation), the velocity error is negligible in Fig. 2.

The protocol presented here will be very welcomed by experimentalists because it clarifies the laboratory conditions necessary for an accurate measurement of the Bohmian velocity, while relaxing the experimental conditions needed for such measurements. As we have emphasized in the introduction, the present work is fully compatible with (and in fact, it is developed within) orthodox quantum mechanics. However, it opens relevant and unexplored possibilities of getting a deeper understanding of many quantum phenomena through the comparison between simulated and measured Bohmian trajectories, instead of using the wave function and its related parameters.

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[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988).
[2] G. J. Pryde, J. L. O'Brien, A. G. White, T. C. Ralph, and H. M. Wiseman, Phys. Rev. Lett. 94, 220405 (2005).
[3] H. M. Wiseman, New Journal of Physics 9, 165 (2007).
[4] O. Hosten and P. Kwiat, Science 319, 787 (2008).
[5] A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D.

Vion, D. Esteve, and A. N. Korotkov, Nature Physics 6, 442447 (2010).
[6] S. Kocsis, B. Braverman, S. Ravets, M. J. Stevens, R. P. Mirin, L. K. Shalm, and A. M. Steinberg, Science 332, 1170 (2011).
[7] A. Matzkin, Phys. Rev. Lett. 109, 150407 (2012).
[8] D. Bohm, Phys. Rev. 85, 166 (1952).
[9] E. Madelung, Zeit. f. Phys. 40, 322 (1927).
[10] D. Dürr, S. Goldstein, and N. N. Zanghí, Journal of Statistical Physics 134, 1023 (2009).
[11] X. Oriols and J. Mompart, Applied Bohmian Mechanics: From Nanoscale Systems to Cosmology (Pan Standford Publishing, 2012).
[12] X. Oriols, Phys. Rev. Lett. 98, 066803 (2007).
[13] A. G. Kofman, S. Ashhab, and F. Nori, Physics Reports (2012), 10.1016/j.physrep.2012.07.001, in press, available on line.
[14] M. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, 2000).
[15] R. Shankar, Principles of quantum mechanics (Plenum Press (New York), 1980).
[16] See supplemental material for details.